Dissipation Properties of Coupled Cavity Arrays

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We propose an approach to analyze the dissipation properties of coupled cavity arrays. Employing a kind of quasi-boson, it is shown that the coupling to a bath renormalizes the localized mode and the interaction between cavities. By virtue of without having to mention the coordinates of bath, this approach would be great conceptual and, moreover, computation advantage. Based on the result, a single-photon transport in the array is examined, and the total transmission rate is presented. Besides, we also suggest a parameter to scale quality of the array.

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Coupled cavity arrays(CCAs), the effective and manipulatable many-body system [1], are known playing a key role in quantum information and quantum devices progressing [2, 3], mimicking and studying strong correlated physics [4–6]. In the term "CCA", "cavity" refers to a region of space within which photon can be efficiently confined, and "coupled" usually implies individual cavities resonantly coupled to each other via the evanescent field. The boost of experimental techniques and advances in theories, over the past years, have paved the way for researching CCA systems more in-depth. CCAs are not only explored in various forms, but also manufactured with smaller mode volume, higher quality factor, more accurate addressability, and increased number, of cavities lying in the set-ups [7, 8]. Moreover, the theoretical proposals, paradigmatically like tight-binding model(TBM) [9, 10], Bose-Hubbard model(BHM) [5, 6, 11], and effective spin model [12, 13], have been put forwarded and predicted a large quantity of novel applications [14–16].

Despite the substantial progress, both in experiment and theory, still many important issues are unsolved. A problem recurs often and in more than one facet is how dissipation would behave in CCA systems [6, 17]. It is always referred to a system, having numerous degrees of-freedom, interacts with a bath, with many, in principle infinitely many, modes. The huge Hilbert space results in great hurdle, even challenge, to fully describe the properties of such system, as yet, there is still lack a method to understand and calculate properly. However, the surge of interest in, so called, quantum manipulation and quantum simulation makes the problem becoming urgent to be solved.

In this paper, we address the issue and show that, for weakly intercavity coupled high-Q array, most experiments in CCAs are carried out such that favour conditions where coupled-mode approximation is valid [18], the problem can be cured by using a kind of quasi-boson picture. Which essence is discarding the coordinates of bath, and describing the system via a effective Hamiltonian. Throughout our discussion, we base on the vacuum, realized, and most general 1D CCA. The reasons are as follows: Firstly, photon leaking from cavity mode is one of the main process of dissipation. Secondly, the approach could also account other sorts of dissipation, like atomic decay, in CCA systems. Finally, and most importantly, it maintains the universality of the proposed approach. According to examine the single photon transport in CCA, the theoretical results are in well agreement with experiments. Which indicates that our work may open up wealth of possibilities for studying dissipative CCA systems conveniently.

We are starting by reviewing the configuration of ideal CCA. As shown in Fig.1(a), N cavities, with a single mode characters by frequency ω_c, are arranged in a period L. The coupling parameter in CCA is mathematically described by a specific overlap integral α. Because individual modes are confined efficiently, only modes of nearest neighbor cavities have a small, but non-vanishing, overlap, denoted by $\alpha\omega_c$. Such configuration is well un-

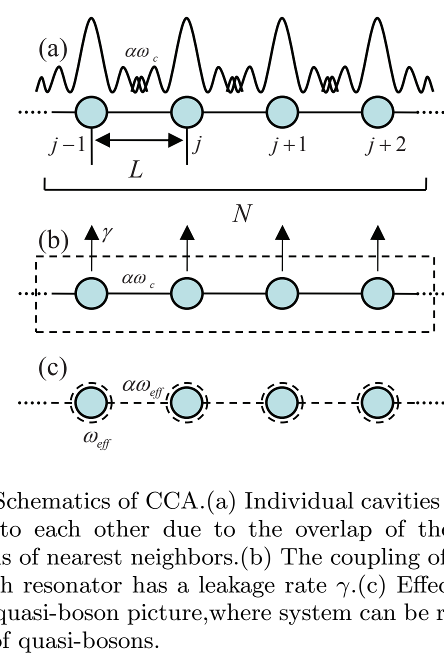

FIG. 1: Schematics of CCA.(a) Individual cavities resonantly coupled to each other due to the overlap of their evanescent fields of nearest neighbors.(b) The coupling of CCA to a bath.Each resonator has a leakage rate γ.(c) Effective treatment in quasi-boson picture,where system can be regarded as a chain of quasi-bosons.

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derstood by using TB Scheme and forms the basis for other CCA systems [1]. In the Coulomb gauge, these localized modes, labelled by φ_j, obey the Maxwell equation [9, 19]:

$$\frac{\epsilon(\mathbf{r})\omega_{\mathbf{c}}^{\mathbf{2}}}{c^2}\varphi_j - \nabla\times(\nabla\times\varphi_j) = 0, \tag{1}$$

and α is given by

$$\alpha = \int dr[\epsilon_0(\mathbf{r}) - \epsilon(\mathbf{r})]\varphi_{\mathbf{j}}^*\varphi_{\mathbf{j+1}}. \tag{2}$$

Where $\mathbf{r}$ is a given, in fact, three dimensional vector, $\epsilon_0(\mathbf{r})$ the dielectric constant of single cavity, and $\epsilon(\mathbf{r})$ the periodic dielectric constant of array.

When N is large enough, periodic boundary condition of CCA is fulfilled and the Hamiltonian (with $\hbar = 1$) reads

$$H_{array} = \omega_c \sum_j c_j^+ c_j - \alpha\omega_c \sum_{\langle j,j'\rangle} c_j^+ c_{j'}. \tag{3}$$

The bosonic operator $c_j^+(c_j)$ creates(annihilates) a excited state at jth cavity, $\sum_{\langle j,j'\rangle}$ sums all pairs of cavities which are nearest neighbors.

As a result of TB interaction, the whole system is no longer a monochromatic field but splits into N resonant modes, which are well explained as liner combination of individual cavity modes, and forms a narrow band in vicinity of ω_c. The spectrum takes the form

$$\omega(k) = \omega_c + 2\alpha\omega_c \cos k_n L \tag{4}$$

and has been observed experimentally by measuring the transmission-phase properties [20]. However, it is worth stressing that the wave vector, $k_n = \frac{n\pi}{N+1}\frac{1}{L}$ for $n = 1$ to N, does not has a direct meaning in terms of photonic momentum but to analog of the lattice vector in solid state physics.

Since we have not added any other features, like Jaynes-Commings interaction and Kerr interaction, the formalism described above is the most common foundation of CCA systems. Therefore, it is advisable to analyze the dissipation properties base on the scheme. In what follows, we will approach the problem in two steps.

Firstly, we consider a single cavity coupled to a bath composed of the infinite set of harmonic oscillators [21]. The bath generally has a continuous spectrum characterized by ω_r and the density of states described by $\rho(\omega_r)$. For simplicity, assuming here only one excited state occupied by either cavity or bath, and the corresponding probability amplitude denoted by e_c and e_r respectively. Besides, because the totality is conservative, one can write the eigenvalue equation as

$$H|\varphi\rangle = \omega|\varphi\rangle, \tag{5a}$$

with

$$H = \omega_c c_0^+ c_0 + \int d\omega_r r^+ r + \int d\omega_r[\eta^*(\omega_r) r^+ c_0 + h.c.], \tag{5b}$$

$$|\varphi\rangle = e_c c_0^+|\emptyset\rangle + \int d\omega_r \rho(\omega_r) e_r r^+|\emptyset\rangle. \tag{5c}$$

$r^+(r)$, which satisfies the commutation relation $[r(\omega_r), r^+(\omega_r')] = \delta(\omega_r - \omega_r')$, creates(destroys) an excited state of bath. $\eta(\omega_r)$ represents the coupling strength between the two, ω the total energy, and $|\emptyset\rangle$ the vacuum state.

Taking the inner product first with $c_0^+|\emptyset\rangle$ and then with $r^+|\emptyset\rangle$ to Eq.(5a),

$$\omega_c e_c + \int d\omega_r \rho(\omega_r)\eta(\omega_r) e_r = \omega e_c, \tag{6a}$$

$$\omega_r e_r + \eta^*(\omega_r) e_c = \omega e_r. \tag{6b}$$

From Eq.(6b), $e_r = \frac{\eta^*(\omega_r)}{\omega-\omega_r} e_c$, and plugging it into Eq.(6a),

$$\omega_c e_c + \int d\omega_r \rho(\omega_r)\frac{|\eta(\omega_r)|^2}{\omega - \omega_r} e_c = \omega e_c. \tag{7}$$

Note that

$$\begin{aligned} &\int d\omega_r \rho(\omega_r)\frac{|\eta(\omega_r)|^2}{\omega - \omega_r} \\ &= \int d\omega_r \rho(\omega_r)\frac{|\eta(\omega_r)|^2}{\omega - \omega_r + i\delta} \\ &= P\int d\omega_r \rho(\omega_r)\frac{|\eta(\omega_r)|^2}{\omega - \omega_r} - i\pi\rho(\omega)|\eta(\omega)|^2 \end{aligned} \tag{8}$$

In the above derivation, we have extended the integration into complex plane and used the relation $\lim\limits_{y\to 0^+} \frac{1}{x+iy} = P\frac{1}{x} - i\pi\delta(x)$, where P denotes the Cauchy principal value, x and y are real variables.

It is reasonable to assume that ω can cause excitation of the cavity sharply peaks around ω_c. Therefore, we can evaluate Eq.(8) at $\omega = \omega_c$,

$$P\int d\omega_r \rho(\omega_r)\frac{|\eta(\omega_r)|^2}{\omega - \omega_r} \approx P\int d\omega_r \rho(\omega_r)\frac{|\eta(\omega_r)|^2}{\omega_c - \omega_r} = \delta\omega_c, \tag{9a}$$

$$i\pi\rho(\omega)|\eta(\omega)|^2 \approx i\pi\rho(\omega_c)|\eta(\omega_c)|^2 = i\gamma. \tag{9b}$$

$\delta\omega_c$ is known analogous to the Lamb shift and significantly small in the case of coupling to surroundings weakly. γ is the decay rate, which indicates a finite lifetime of cavity mode.

Thus Eq.(7) becomes

$$(\omega_c + \delta\omega_c - i\gamma)e_c = \omega e_c. \tag{10}$$

Which means that due to the coupling, the cavity mode is renormalized by reckoning in frequency shift and intrinsic loss. The above expression is equivalent to Eqs.(6a) and (6b), however, does not contain degrees of freedom of bath. It motivates us to introduce a quasi-boson described by b and having a complex eigenfrequency

$\omega_{eff} = \omega_c - i\gamma$, where $\delta\omega_c$ has been absorbed into ω_c, to redescribe the cavity mode. And then rephrasing Eqs.(5a)-(5c),

$$H_{eff}|\varphi\rangle = \omega_{eff}|\varphi\rangle, \tag{11}$$

with the effective Hamiltonian $H_{eff} = \omega_{eff} b^+ b$ and now $|\varphi\rangle = e_c b^+|\emptyset\rangle$ referred to as quasinormal-mode [22]. Because of loss energy, the system would nonconservative, and the corresponding operators non-Hermitian. To compare the two descriptions, the communication relation of b reads $[b, b^+] = 1 + i\frac{2\gamma}{\omega_c}$. Clearly, $\frac{2\gamma}{\omega_c}$ in order of $\frac{1}{Q}$, thus bosonic communication relation is approximately satisfied.

Then next, we return to the case, see Fig.1(b), CCA coupled to a bath and each resonator has a leakage rate γ. It is verified experimentally the main sources of loss are individual cavities, while the additional loss caused by periodic structure is negligible [8]. Combining the characteristic cavities are weakly coupled, such system can be regarded as a chain of quasi-bosons, see Fig.1(c), and mapped safety onto TB scheme. According to Eq.(1), the associated eigenmodes, labelled by ψ_j, satisfy

$$\frac{\epsilon(\mathbf{r})}{c^2}(\omega_c^2 + \gamma^2)\psi_j - \nabla \times (\nabla \times \psi_j) = 0. \tag{12}$$

For γ^2 is Q^2 orders of magnitude smaller than ω_c^2, the minimal loss on each lattice site does not generate noticeable alteration to localized modes. Which also illustrates that quasi-boson picture is an excellent approximation to the established mode.

Consequently, the relevant overlap integral, α', is given by

$$\begin{aligned}\alpha' &= \int dr[\epsilon_0(\mathbf{r}) - \epsilon(\mathbf{r})]\psi_{\mathbf{j}}^* \psi_{\mathbf{j+1}} \\ &\approx \int dr[\epsilon_0(\mathbf{r}) - \epsilon(\mathbf{r})]\varphi_{\mathbf{j}}^* \varphi_{\mathbf{j+1}} \\ &= \alpha.\end{aligned} \tag{13}$$

Hence, we reach the familiar Hamiltonian but take dissipation into account,

$$H = \omega_{eff}\sum_j b_j^+ b_j - \alpha\omega_{eff}\sum_{\langle j,j'\rangle} b_j^+ b_{j'}. \tag{14}$$

Yet interestingly, without having to mention the external degrees of freedom, the effective treatment would be of great conceptual and, moreover, computational advantage rather than treatment of universe [17]. One key feature is now the loss seems owing to the nonideal boundary but not field oscillation, viz described by a constant but not operators. In addition, it should also to point out that the specific impact of renormalization to interaction terms may vary from case to case, nevertheless all of those represented by a small quantity $\alpha\gamma$. This is consistent with the conditions discussed previous.

To demonstrate the validity of our approach, we consider now the single-photon transport in the CCA. In the simplest possible context, we assume that a photon has somehow been inject into 1st cavity and propagating to the right. The frequency, ω, of photon satisfy dispersion relation (4), thus photon hopping can occur between neighboring cavities due to the overlap of the light modes. So the problem we treated can be described by Hamiltonian (14). Furthermore, to focus on the total transmission rate, we can restrict us to solve the stationary Schödinger equation

$$H|\psi\rangle = \omega|\psi\rangle, \tag{15}$$

with $\psi = \sum_j e_j b_j^+|\emptyset\rangle$, and take [23, 24] $e_j = \begin{cases} e_{j^-} = e^{ik_n sL} + r_j e^{-ik_n sL} & s < j \\ e_{j^+} = t_j e^{ik_n sL} & s > j \end{cases}$, $s = 1$ to N. Where r_j and t_j denote the local transmission amplitude and reflection amplitude of photon respectively. Solving Eq.(15) by using the continuous condition $e_{j^-} = e_{j^+}$ at jth site and the constraint condition$|r_j|^2 + |t_j|^2 \leq 1$ due to the irreversible loss of energy, we get

$$r_j = \frac{\kappa\cos k_n L - \gamma}{(\gamma + \xi|\sin k_n L| - \kappa\cos k_n L) - i\kappa|\sin k_n L|} e^{i2k_n jL}, \tag{16a}$$

$$t_j = \frac{(\xi - i\kappa)|\sin k_n L|}{(\gamma + \xi|\sin k_n L| - \kappa\cos k_n L) - i\kappa|\sin k_n L|}. \tag{16b}$$

Above, $\xi = 2\alpha\omega_c$ and $\kappa = 2\alpha\gamma$ for compactness, $e^{i2k_n jL}$ is position-dependent global phase but does not affect the transport properties, and the absolute value sign is need for energy conservation.

Before proceeding, here we briefly outline some of the main features of r_j and t_j. The nonzero reflection amplitude is caused by local loss. Under the circumstance of system is confined in one dimension, incoming photon having possibility to escape toward the opposite direction. Note however, this possibility would not make photon enters the previous cavity and becomes left-moving photon, but eventually decay to other dimensions. Local loss also leads to the nonunitary transmission amplitude. By dropping the second-order small quantity κ, the maximum of transmission coefficient approximates $\frac{1}{(1+\gamma/\xi)^2}$, which means the local transport properties is determined by the competition between photon hopping and decay, since they are the only channels photon can leave a certain cavity.

And then, the total transmission rate, T, can be intuitively written as

$$T = \prod_j |t_j|^2 = |t_j|^{2N}. \tag{17}$$

The transmission spectrum, shown in Fig.2(a), retains the symmetry of dispersion relation (4) and vanishes at band edges. When the propagating photon is on resonant with individual cavity, $\omega = \omega_c$, the spectrum exhibits the

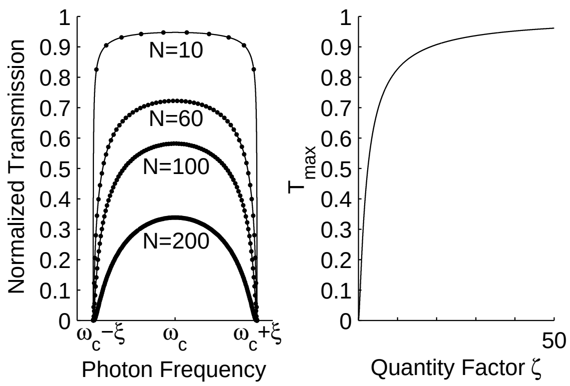

FIG. 2: (a) Transmission spectrum for single-photon transport in CCA. The total transmission rate of each splitted resonant mode is denoted by dots and fitted by a curve. Following the experimental parameters in Ref. [8], $Q = 1.1\times10^6$ and $\xi = 6.47\times10^{-4}\omega_c$, the output power, concretely like when $N = 60$, is well agreement. (b) Dependence of the maximal transmission rate on quantity factor ζ. In Fig.(2a), ζ equal to 71.17, 11.86, 7.117, and 3.559 respectively.

maximum, $T_{max} \approx \frac{1}{(1+\gamma/\xi)^{2N}}$. While the ratio between local loss rate and intercavity coupling strength is far less than one, we can take $(1+\frac{\gamma}{\xi})^{2N} = 1 + \frac{2N\gamma}{\xi} + \cdots$, thus

$$T_{max} \approx \frac{1}{(1+N\gamma/\xi)^2} = \frac{1}{(1+N/\alpha Q)^2}. \quad (18)$$

By substituting $Q = \frac{2\omega_c}{\gamma}$ and $\xi = 2\alpha\omega_c$, the maximal transmission rate now is described directly with three essential parameters of CCA. It is helpful to define a new quality factor, $\zeta = \frac{\alpha Q}{N}$, to scale CCA's transport properties, which lead to

$$T_{max} = \frac{1}{(1+\zeta)^2}. \quad (19)$$

Furthermore, the transport loss mainly stems from cavity-mode decay, thus ζ could reflect as well as dissipation properties for other CCA systems. A high-ζ array, see Fig.1(a) for example, is often referred to steep or sharp spectrums.

In summary, to aim at the descriptive difficulty caused by the coupling of CCA systems to environment, we have proposed a kind of quasi-boson picture and shown its effectiveness by analyzing the single-photon transport. Here we would like to emphasize the generality of our approach, which is capable of treating dynamical problems and other sorts of dissipation [24, 25] and provides a starting point for discussing more complicated situations [1].

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[1] M. J. Hartmann, F. G. S. L. Brandão, and M. B. Plenio, Laser Photonic Rev. **2**, 527 (2008).
[2] T. Baba, Nature Photon. **2**, 465 (2008).
[3] T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. OBrien, Nature(London) **464**, 45 (2010).
[4] M. J. Hartmann, F. G. S. L. Brandão, and M. B. Plenio, Nature Phys. **2**, 849 (2006).
[5] A. D. Greentree, C. Tahan, J. H. Cole, and L. C. L. Hollenberg, Nature Phys. **2**, 856 (2006); M. I. Makin, J. H. Cole, C. Tahan, L. C. L. Hollenberg, and A. D. Greentree, Phys. Rev. A **77**, 053819 (2008).
[6] D. Rossini and P. Fazio, Phys. Rev. Lett. **99**, 186401 (2007).
[7] F. Xia, L. Sekaric, and Y. Vlasov, Nature Photon. **1**, 65 (2007).
[8] M. Notomi, E. Kuramochi, and T. Tanabe, Nature Photon. **2**, 741 (2008).
[9] A. Yariv, Y. Xu, R. K. Lee, and A. Scherer, Opt. Lett. **24**, 711 (1999).
[10] L. Zhou, Y. B. Gao, Z. Song, and C. P. Sun, Phys. Rev. A **77**, 013831 (2008).
[11] M. Knap, E. Arrigoni, and W. von der Linden, Phys. Rev. B **81**, 104303 (2010).
[12] D. G. Angelakis, M. F. Santos, and S. Bose, Phys. Rev. A **76**, 031805(R) (2007).
[13] A.-C. Ji, X. C. Xie, and W. M. Liu, Phys. Rev. Lett. **99**, 183602 (2007).
[14] M. F. Yanik and S. Fan, Phys. Rev. Lett. **92**, 083901 (2004);J.-T. Shen and S. Fan, *ibid.* **95**, 213001 (2005).
[15] C.P. Sun, L. F. Wei, Yu-xi Liu, and F. Nori, Phys. Rev. A **73**, 022318 (2006).
[16] P. Longo, P. Schmitteckert, and K. Busch, Phys. Rev. Lett. **104**, 023602 (2010).
[17] M. H. Szymańska, J. Keeling, and P. B. Littlewood, Phys. Rev. Lett. **96**, 230602 (2006).
[18] F. Lederer, G. I. Stegeman, D. N. Chritodoulides, G. Assanto, M. Segev, and Y. Silberberg, Phys. Rep. **463**, 1 (2008).
[19] R. J. Glauber and M. Lewenstein, Phys. Rev. A **43**, 467 (1991).
[20] M. Bayindir, B. Temelkuran, and E. Ozbay, Phys. Rev. Lett. 84, 2140 (2000);*ibid.* Phys. Rev. B. **61**, R11855 (2000).
[21] P. Lambropoulos and D. Petrosyan, *Fundamentals of Quantum Optics and Quantum Information* (Springer-Verlag, Berlin, 2007).
[22] P. T. Leung, S. Y. Liu, and K. Yong, Phys. Rev. A **49**, 3057 (1994);E. S. C. Ching, P. T. Leung, A. M. van den Brink, W. M. Suen, S. S. Tong, and K. Young, Rev. Mod. Phys. **70**, 1545 (1998).
[23] Y. Xu, Y. Li, R. K. Lee, and A. Yariv, Phys. Rev. E **62**, 7389 (2000).
[24] J.-T. Shen and S. Fan, Phys. Rev. A **79**, 023837 (2009).
[25] D. Dalidovich and M. P. Kennett, Phys. Rev. A **79**, 053611 (2009).